XXIII. *An account of Experiments on the velocity of Sound, made in Holland.* By Dr. G. Moll, *Professor of Natural Philosophy in the University of Utrecht, and* Dr. A. Van Beek.

Read March 18, 1824.

Sir Isaac Newton's formula, expressing the velocity of sound,

$$c = \sqrt{\frac{gp}{D}}$$

has since his time been investigated and demonstrated by several first-rate mathematicians. Actual experiments however on this velocity, instituted in various countries, and under different circumstances, went to prove that the celerity of sound, found by experimiment, is about one-sixth greater than can be deduced by theory.

The celebrated Laplace accounted for this difference between experiment and theory, by showing that it could be attributed to the heat evolved by the compression of the particles of air which is effected by the undulations of sound. It was found impossible to determine the quantity of heat thus evolved, by the compression which sound occasions in the particles of the air; and therefore it was deemed expedient to multiply Sir Isaac Newton's formula by a constant factor $\sqrt{1 + k}$, the value of which was determined by experiment. Sir Isaac's formula thus altered, became

$$c = \sqrt{\frac{pg}{D}} \cdot \sqrt{1 + k}.$$

Thus, by the experiments of the French Academicians of

1738, the most accurate on this subject of that time, the value of k was found equal to 0,4254. It is plain that this correction of the original formula is merely empirical, and dependant on the accuracy of experiments, which in 1738, had certainly not attained the perfection which is required at present.

In consequence, this formula was thus altered by LAPLACE,

$$\sqrt{\frac{\sqrt{g\,p}}{D}} \cdot \sqrt{\frac{c'}{c}};$$

in which c' is the specific heat of the air under a constant pressure, and c is the specific heat of the air under a constant volume.*

My friend Dr. VAN REES, Professor in the University of Liege, gave a demonstration of this correction $\sqrt{\frac{c'}{c}}$, which will be subjoined to the present paper,† and which may be compared with that of Mr. POISSON.‡ The value of $\frac{c'}{c}$ was determined by LAPLACE from experiments of Messrs. LAROCHE and BERARD,§ and found equal to 1,4954; but later and more accurate experiments of Messrs. GAY LUSSAC and WELTER brought it to 1,3748.

Another cause of the difference between actual experiments on the velocity of sound and its theory, exists in the variable force of the wind, which either accelerates or retards the velocity of sound, according to the direction from which it is blowing. It appears that this cause of error may be annihilated in the following manner. Let sounds be ex-

* LAPLACE in Ann. de Phys. et Chim. Tom. iii. p. 238.

† Dissertatio de celeritate soni, Trajet. 1818.

‡ Annales de Phys. et de Chim. Mai 1823, d. 5.

§ Ibid. Annales de Chimie, Tom. 85, p. 72.

cited exactly at the same time on both ends of a basis, and let two observers stationed on these ends, measure the velocity with which sounds travels from one end of the basis to the other. It is quite clear, that the action of the wind must necessarily accelerate the velocity of the sound excited at one end of the basis, as much as it will retard that at the other end, and thus the medium of these two velocities will give the velocity in tranquil air. This method was not adopted by the French Academicians of 1738, in their experiments between Monthlery and Montmartre. Cannon was fired at one of these stations, whilst the observers were at the other, and thus the results remained affected by the whole effect of the wind. It was found expedient therefore to repeat these experiments with more accuracy, and this was executed with great precaution on Mr. LAPLACE's proposal, by Messrs. ARAGO, PRONY, MATHIEU, BOUVARD, HUMBOLDT, and GAY LUSSAC. The experiments were made in 1822, on the basis of Monthlery and Villejuif. In two successive days, the 21st and 22d of June, 1822, seven shots were fired on both stations, and observed on the other; the difference of time in which the corresponding shots were fired at both stations not exceeding five minutes, and from these seven corresponding shots the result was deduced.

These experiments having never been made in this country with any thing like sufficient accuracy, His Royal Highness PRINCE FREDERICK, second son of HIS MAJESTY the KING of the NETHERLANDS, and Master General of the Ordnance, was pleased to approve of our proposal of repeating the same, and to authorise Lieutenant-Colonel KUYTENBROUWER, and the officers and men of the battalion of Artillery under

his command, to give us every assistance in their power, and to take an actual part in these experiments.

SECTION II.

As fitted places to make these experiments, two elevated spots were selected on the extended heaths of the Province of Utrecht. One of these is a small hill between the town of Naarden and the village of Blaricum, and called the *Kooltjesberg*; the other is somewhat higher, and situated on the right of the road from Utrecht to Amersfoort, and very near the last town. Both places were distinctly visible from one another, and the distance was between 17000 and 18000 metres (9664 fathoms). Our time was kept by two time-keepers, which the Minister of Marine had kindly furnished us with; one made by ARNOLD, the other by our countryman Mr. KNEBEL. But the exact interval between the observation of light, and the perception of sound, and consequently the velocity of sound, was measured with small clocks with conical pendulums. They are made at Wesel by Mr. WILHELM PFAFFIUS, and proved remarkably well adapted for this purpose. It is well known that HUIGENS laid down the properties of the conical, or centrifugal pendulum, but if we are not mistaken, they were employed for similar purposes for the first time by the German philosopher BENZENBERG.* These clocks with a conical pendulum divide the 24 hours of the day in 10,000,000 parts, and one of the indexes gives $\frac{1}{100}$ part of a decimal second. This index or second hand remains quiet, whilst the watch work continues moving as long as a certain spring is not pressed down with the finger; and on removing the

* Some account of these clocks is given in GILBERT's Annalen d. Physik, 1804, B. 16, p. 494; and New Series, vol. v. p. 333.

finger, the index is reduced to rest in the identical moment. Thus the index being at o, the spring is pressed down by the observer at the very instant the light of the opposite station is observed; the index continues moving till the report of the shot is heard, when the finger is withdrawn, and the index stopped instantaneously. The number of turns and fractions of a turn of the index shows the time elapsed between the fire and the report. There was a conical or centrifugal clock on each station; besides these, each station was furnished with a good barometer, carefully compared with a standard barometer of Mr. Dollond, several good thermometers made by Messrs. Dollond and Newman, besides a sufficient number of excellent telescopes of Dollond's, and so placed on stands adapted for the object as to bring the opposite station without trouble in the field of the telescope. The moisture of the air was determined for the first time in such experiments by Mr. Daniell's hygrometer, one of which was placed at each station. The direction of the wind was determined by very good vanes contrived by the Artillery officers. At each station a twelve pounder and a six pounder was planted, and the instruments were disposed in, or in the vicinity of tents erected for the purpose. Professor Moll, with Lieutenants Renault and Dilg, was stationed at the *Kooltjesberg*. Dr. Van Beek, with Lieutenants Sommerton, Van Den Bylaardt and Seelig were on the other station, which is commonly called *Zevenboompjes*, or seven trees, from the circumstance of seven trees being planted on this lonely elevation. Several gentlemen cadets of the Artillery, and several students of the University, were at both stations employed in observing the different instruments.

The barometers and thermometers were of course ob-

served in the open air; Mr. DANIELL's hygrometers were also placed in the open air; and the light of a candle reflected from the surface of the ball of the hygrometer, gave the means of observing the deposition of dew with great accuracy.

It was deemed of the utmost importance that the shots on both stations should be fired at as nearly the same moment as possible. To obtain this, the following plan was adopted. At 7^h $55'$ P. M. by the chronometer of Zevenboompjes, a rocket was fired at Zevenboompjes, which being observed at the other station of *Kooltjesberg*, was immediately answered by another rocket from the latter place. This was the signal that on both stations every thing was ready for observation. At 8^h $0'$ $0''$ by the chronometer of *Zevenboompjes*, a cannon shot was fired on that station, whilst the observers at Kooltjesberg took as exactly as possible the time on their chronometer when the light was observed. A second shot was fired at Zevenboompjes at 8^h $5'$ $0''$ P. M. by the chronometer of that station, and the time at which the light was seen was carefully taken down by the chronometer of Kooltjesberg. By these means the difference of the two chronometers at both stations, in a distance of about 9 miles, was ascertained with great accuracy; and in order to show that this preparatory investigation was made with due care, a cannon shot was fired on both places at the moment when the chronometer of Zevenboompjes marked 8^h $10'$ $0''$. If the lights of both shots were seen exactly at the same time, it was a proof that the difference of both time-keepers was known, and that experiments might be safely made.

We own that we did not suppose before hand, that it could

be possible to fire continually guns at a distance of 9 miles exactly at the same second; but the very great attention and ability of our artillery men overcame this difficulty. Between our shots at the two stations there was never a greater difference than 1″ or 2″, whilst this difference in the experiments of the French philosophers of 1822, went to 5 minutes. This exact correspondance in the firing of the guns was obtained in the following manner. At each station an officer had the chronometer placed before him on a small table very near the gun; a non-commissioned officer or gentleman cadet stood ready with the port fire near the touch-hole; and at the instant required the officer holding the chronometer pressed the arm of the person who was to fire the gun, which went off at the very moment. With a little practice they were certain to fire the gun at any given second.

The first night of our experiments, the 23d, 24th, and 25th of January, 1823, we experienced the same annoyance of which the French philosophers had to complain the first night of theirs. The report of the shots of Zevenboompjes was not heard at all at the station of Kooltjesberg. But at Zevenboompjes all the shots of Kooltjesberg were distinctly heard. After the first night we constantly used the metal twelve pounders loaded with 6lbs. of gunpowder. The 26th of January all the shots were heard at Kooltjesberg, but none were perceived at the opposite place. But the wind shifting the following night, a good number of corresponding or simultaneous shots were distinctly heard on both stations. The particulars of the experiments made in these different days will be found in the tables annexed to this paper. The dis-

appointment we met with on the first days was however not entirely fruitless; we were convinced by it, that none but exactly corresponding shots can be of use in determining the velocity of sound. The result of the observations of 25th and 26th of January, when the reports were heard at one station only, and reduced to 0° temperature of the centigrade scale, and dry air, give differences of $\frac{1}{53}$, whilst the observations of 27th and 28th of January, when shots were distinctly heard on both stations, had only a difference of $\frac{1}{502}$ from each other.

The time which sound employs to travel from one station to another being duly ascertained, we proceeded to measure the distance between both stations. The distances of the steeples of Utrecht and Amersfoort, Utrecht and Naarden, Naarden and Amersfoort being accurately known, we measured angles on our stations between these steeples, and on each steeple between the other steeples and the stations. Thus the distance was calculated by four different triangles, and the greatest difference between these calculations was $2^{m},45$ or 8 feet, which appeared of no consequence in these experiments. The distances of the different steeples which we took for our basis, result from the very exact geometrical survey of General KRAYENHOFF.*

From these different data we found, by calculations of which the detail will be given hereafter, that in our experiments at a temperature of 32° FAHRENHEIT, or 0° of the centigrade scale, the velocity of sound is $332^{m},049$, or 1089,7445 English feet per sexagesimal second. A table showing the

* Précis des Opérations Géodésiques et Trigonométriques en Hollande, par le Général KRAYENHOFF.

comparison of our experiments, with the observations of other philosophers, is also annexed to this paper.

SECTION III.

Observations to ascertain the distance between the stations of Kooltjesberg and Zevenboompjes.

A. Mensuration of angles with a ten-inch Repeating Circle, made by LENOIR, *in Paris.*

a. Angles measured on the steeple of the Cathedral of Utrecht.

α. Between Kooltjesberg and Amersfoort.

Date.	No.	Observations.	Repeated Angles.	Single Angles.	
			° ′ ″	° ′ ″	
1823. 1 Aug. 3ʰ P.M.	1.	twice	98 22 30	49 11 15	Mean angle 49° 11′ 18″,2. Reduced to the center of station 49° 10′ 25″,2.
	2.	4	196 45	49 11 15	
	3.	6	295 9	49 11 20	
	4.	8	393 31 30	49 11 26	
	5.	10	491 52 30	49 11 15	

The reduction to the centre of station has been made by the well known formula $C = O + \frac{r \text{ Sin. } (O+y)}{D \text{ Sin. } 1''} - \frac{r \text{ Sin. } y}{G \text{ Sin. } 1''}$ taken from DELAMBRE's base du Systême Métrique, et PUISSANT's Traité de Géodésie.

β. Angles measured at Utrecht between Kooltjesberg and 7 Boompjes.

Date.	No.	Observations.	Repeated Angles.	Single Angles.	
			° ′ ″	° ′ ″	
1823. 1st. Aug. 3ʰ P.M.	1.	Double	103 2 30	51 31 15	Mean angle 51° 31′ 9″,1. Reduced to center of station 51° 30′ 14″,1.
	2.	Quadruple	206 5	51 31 15	
	3.	Sextuple	309 6 30	51 31 5	
	4.	Octuple	412 9	51 31 7,5	
	5.	Decuple	515 10 30	51 31 3	

γ. Angles measured at Utrecht between Amersfoort and the Pyramid of the Camp of Zeist.

Date.	No.	Multiple Angles.	Repeated Angles.	Single Angles.
			° ′ ″	° ′ ″
1823. 1st. Aug. 3h P. M.	1.	Double	43 17 30	21 38 45
	2.	Quadruple	86 35	21 38 45
	3.	Sextuple	129 51	21 38 30
	4.	Octuple	173 8	21 38 30
	5.	Decuple	216 27 30	21 38 45

Mean angle 21° 38′ 39″; reduced to centre of station 21″ 38′ 0″,5.

b. Angles taken on tte pyramid of the Camp of Zeist between Utrecht and Amersfoort.

Date.	No.	Multiples.	Repeated Angles.	Single Angles.
			° ′ ″	° ′ ″
1823. 7th Aug. 8h A. M.	1.	Double	225 50	112 40
	2.	Quadruple	450 41 30	112 40 22,5
	8.	Sextuple	676 1 30	112 40 22,5
	4.	Octuple	901 21 30	112 40 22,5
	5.	Decuple	1126 45 30	112 40 33

Mean angle 112° 40′ 20″,1; angle taken from centre of station.

β. Between Utrecht and Zevenboompjes.

Date.	No.	Multiples.	Repeated Angles.	Single Angles.
			° ′ ″	° ′ ″
1823. 7th Aug. 8h A. M.	1.	Double	209 35	104 47 30
	2.	Quadruple	419 9 30	104 47 22,5
	3.	Sextuple	628 45	104 47 30
	4.	Octuple	838 18	104 47 15
	5.	Decuple	1047 53	104 47 18

Mean angle 104° 47′ 23″; angle taken from centre of station.

c. Angles taken on the Station of Zevenboompjes.

α. Between Utrecht and Kooltjesberg.

Date.	No.	Multiples.	Repeated Angles.	Single Angles.
			° ′ ″	° ′ ″
1823. 7th Aug. 3h P. M.	1.	Double	153 33 30	76 46 45
	2.	Quadruple	307 6 30	76 46 37,5
	3.	Sextuple	460 39	76 46 30
	4.	Octuple	614 12	76 46 30
	5.	Decuple	767 44	76 46 24

Mean angle 76° 46′ 33″.3; angle taken from centre of station.

β. Between Utrecht and Pyramid.

Date.	No.	Multiples.	Repeated Angles.	Single Angles.	
			° ′ ″	° ′ ″	
1823. 8th Aug. 12h	1.	Double	111 47	55 53 30	Mean angle 55° 53′ 19″,2; angle taken from centre of station.
	2.	Quadruple	223 33	55 53 15	
	3.	Sextuple	335 20	55 53 20	
	4.	Octuple	447 6 30	55 53 19	
	5.	Decuple	558 52	55 53 12	

γ. Between Kooltjesberg and Amersfoort.

Date.	No.	Multiples.	Repeated Angles.	Single Angles.	
			° ′ ″	° ′ ″	
1823. 8th Aug. 12h	1.	Double	159 34 30	79 47 15	Mean angle 79° 47′ 16″,1; taken from centre of station.
	2.	Quadruple	319 8	79 47	
	3.	Sextuple	478 44 30	79 47 25	
	4.	Octuple	638 19	79 47 22,5	
	5.	Decuple	797 53	79 47 18	

c. Angles taken on the Steeple of Amersfoort.

α. Between Utrecht and Kooltjesberg.

Date.	No.	Multiples.	Repeated Angles.	Single Angles.	
			° ′ ″	° ′ ″	
1823. 7th Aug.	1.	Double	145 16	72 38	Mean angle 72° 38′ 14″,7; angle reduced to centre of station 72° 37′ 7″,2.
	2.	Quadruple	290 32 30	72 38 7,5	
	3.	Sextuple	435 50	72 38 20	
	4.	Octuple	581 6 30	72 38 19	
	5.	Decuple	726 24 30	72 38 27	

β. Between Utrecht and the Pyramid.

Date.	No.	Multiples.	Repeated Angles.	Single Angle.	
			° ′ ″	° ′ ″	
1823. 7th Aug.	1.	Double	91 27	45 43 30	Mean angle 45° 43 18″,8; reduced to centre of station 45° 41′ 32″2.
	2.	Quadruple	182 53	45 43 15	
	3.	Sextuple	274 20 30	45 43 25	
	4.	Octuple	365 46	45 43 15	
	5.	Decuple	457 11 30	45 43 9	

γ. *Between Kooltjesberg and Zevenboompjes.*

Date.	No.	Multiples.	Repeated Angles.	Single Angle.
			° ′ ″	
1723. 7th Aug.	1.	Double	187 42 30	93 51 15
	2.	Quadruple	375 27	93 51 45
	3.	Sextuple	563 11 30	93 51 55
	4.	Octuple	750 45	93 50 37
	5.	Decuple	938 38	93 51 48
	7.	Duodecuple	1126 28	93 52 20
	7.	Quatuordecuple	1314 6 30	93 51 53

Mean angle 93° 51′ 39″; reduced to centre of station 93° 45′ 7″

a. Angles taken at Kooltjesberg.

α. *Between the Steeples of Utrecht and Amersfoort.*

Date.	No.	Multiples.	Repeated Angles.	Single Angles.
			° ′ ″	° ′ ″
1823. 8th Aug.	1.	Double	116 25 30	58 12 45
	2.	Quadruple	232 51	58 12 45
	3.	Sextupie	349 15 30	58 12 35
	4.	Octuple	465 41	58 12 37
	5.	Decuple	582 6 30	58 12 39

Mean angle 58° 12′ 40″,3, from centre of station.

β. *Between the Steeples of Utrecht and Naarden.*

Date.	No.	Multiples.	Repeated Angles.	Single Angles.
			° ′ ″	° ′ ″
1823. 8th Aug.	1.	Double	191 27	95 43 30
	2.	Quadruple	382 54	95 43 30
	3.	Sextuple	574 20 30	95 43 25
	4.	Octuple	765 47	95 43 22
	5.	Decuple	957 13 30	95 43 21

Mean angle 95° 43′ 25″,7, from centre of station.

b. Between the Steeples of Utrecht and Zevenboompjes.

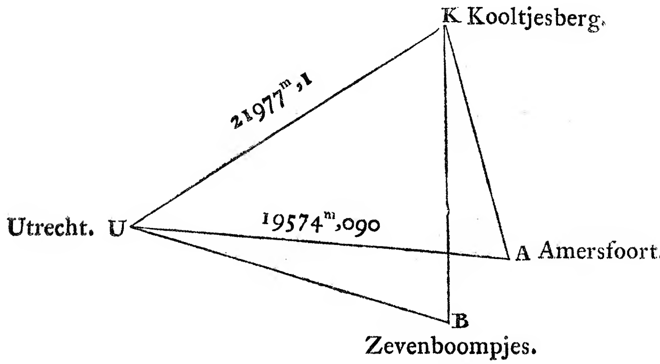
Date.	No.	Multiples.	Repeated Angles.	Single Angles.
			° ′ ″	
1823. 8th Aug.	1.	Double	103 27 30	51 43 45
	2.	Quadruple	206 55	51 43 45
	3	Sextuple	310 22	51 43 40
	4.	Octuple	413 50	51 43 45
	5.	Decuple	517 16 30	51 43 39

Mean angle 51° 43′ 42″,8, from centre of station.

SECTION IV.

Calculation of the distance between the stations of Kooltjesberg and Zevenboompjes, being the two points where the experiments on the velocity of sound were made.

a. First triangle.



The distance between Utrecht and Amerfoort Steeples from General KRAYENHOFF's Survey is $19574^{m},090$.

	°	′	″	
Angle UAK =	72	37	7,2	by mensuration.
AKU =	58	12	40,3	
AUK =	49	10	25,2	
	180	0	12,7	

Angle UAK =	72	37	2,96
AKU =	58	12	36,07
AUK =	49	10	20,97

calculating with these angles and UA, the length UK is found equal to $21977^{m},1$.

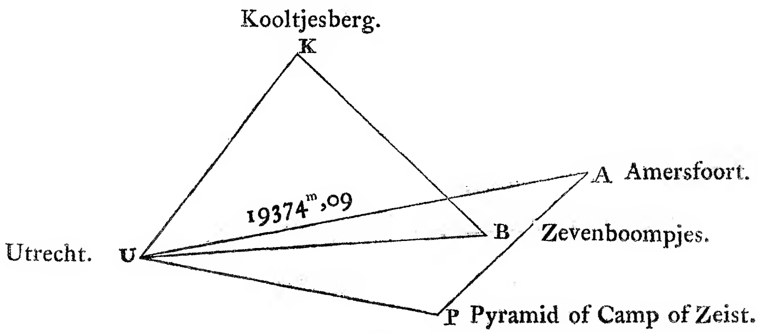
b.

In triangle UBK, we have now UK $= 21977^{m},1$.

	°	′	″
U B K =	76	46	33,3
B U K =	51	43	42,8
B U K =	51	30	14,1
	180	0	30,2
Thus, U B K =	76	46	23,24
B K U =	51	43	32,73
B U K =	51	30	4,03

With these data we have the distance of the two stations of Kooltjesberg and Zevenboompjes 17668,4 metres.

b. Second triangle.



The distance UA between Utrecht and Amersfoort taken from the Survey of General Krayenhoff.

	°	′	″	
U A P =	45	41	32,2	measured angles.
A U P =	21	38	0,5	
U P A =	112	40	20,1	
	179	59	52,8	
Thus U A P =	45	41	34,6	
A U P =	21	38	2,9	
U P A =	112	40	22,5	

Calculating UP the distance from Utrecht to the Pyramid of the Camp of Zeist, we have UP = 15180^{m},5 metres.

Again, in triangle U B P we have UP = 15180,5 metres.

< U B P = 55° 53′ 19″,2	by mensuration for	< K U B = 51° 30′ 14″,1	by mensuration.
U P B = 104 47 23, 1		K U A = 49 10 25, 2	
P U B = 19 18 11, 6		Substract. 2 19 48, 9	
179 58 53, 9		< A U P = 21 38 0, 5	
		< P U B = 19 18 11, 6	

Thus we shall have

$$
\begin{aligned}
UBP &= 55^{\circ}\ 53'\ 41'',23\\
UPB &= 104\ \ 47\ \ 45,14\\
PUB &= 19\ \ 18\ \ 33,63
\end{aligned}
$$

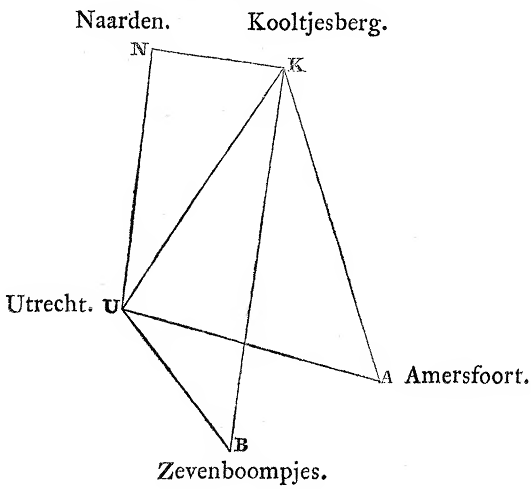
Calculating with these data UB, we have it equal to 17725,8 metres.

But in $\triangle$ U B K, we have

$$
\left.\begin{aligned}
UBK &= 76^{\circ}\ 46'\ 23'',24\\
BKU &= 51\ \ 43\ \ 32,73\\
BUK &= 51\ \ 30\ \ \ 4,03
\end{aligned}\right\}\text{found by calculation above.}
$$

Therefore we find B K = 17670,85 metres.

c. Third triangle.



In the △ U K N

U N = 22987^{m},369 metres by General KRAYENHOFF's Survey.

The angle A U N = 61° 23′ 56″,324, measured by General KRAYENHOFF,

	°	′	″	
A U K =	49	10	25,200	
∠ K U N =	12	13	31,124	
∠ U K N =	95	43	25, 7	
∠ U N K =	72	3	3, 2,	concluded angle.

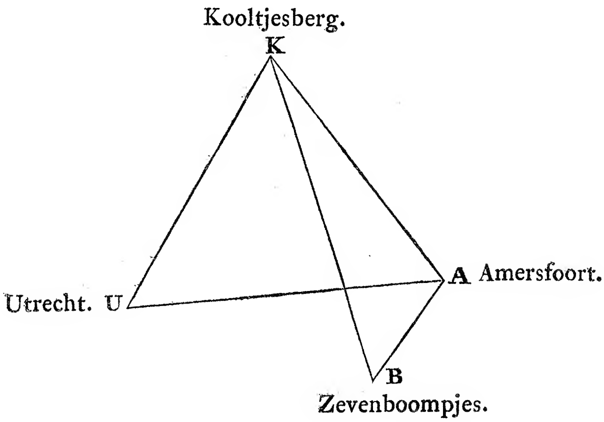
Now calculating U K we have it 21978,2 metres.

Further, in triangle U B K, we have

	°	′	″	
U B K =	76	46	23,24	by calculation above-mentioned.
B K U =	51	43	32,73	
B U K =	51	30	4,03	

This gives B K = 17669,3 metres.

d. Fourth triangle.



In this triangle, U A K, the distance U A from Utrecht to Amersfoort, is known from General KRAYENHOFF's Survey, 19574,090 metres.

$$\angle UAK = 72^\circ\ 37'\ 2'',96$$
$$AKU = 58\ \ 12\ \ 36,07$$
$$AUK = 49\ \ 10\ \ 20,97$$

By these data AK is found equal to 17425,4 metres, the distance from Amersfoort to Kooltjesberg.

In the Δ KBA, we have AK = 17425,4 metres.

$$\angle KBA = 79^\circ\ 47'\ 16'',1$$
$$\angle BAK = 93\ \ 45\ \ 7$$
$$\angle AKB = 6\ \ 28\ \ 57,5$$
$$180\ \ 1\ \ 20,6$$

Thus,

$$KBA = 79\ \ 46\ \ 49,2$$
$$BAK = 93\ \ 44\ \ 40,2$$
$$AKB = 6\ \ 28\ \ 30,6$$

With these data calculating BK, we have the distance between both stations 17668,55.

By these four different methods of calculation we have the distance between the two points when the observations were made, viz. from Kooltjesberg to Zevenboompjes.

	Metres.
By the 1st method .	17668,40
2d . . .	17670,85
3d . . .	17669,30
4th . . .	17668,55

Mean distance of basis 17669,28 metres, or 9664,7044 English fathoms, the metre being rated at 39,3824 English inches.

SECTION V.

Rate of Clocks with conical Pendulums.

Having shown in the preceding section how the distance between the two stations was ascertained, we must now proceed to show that the clocks with conical pendulums, with which the interval of time between the light and the report was measured, kept a regular rate; for this purpose they were frequently compared on each station with the box chronometers. The following table shows how many decimal seconds and fractions of seconds passed on the conical centrifugal pendulum clock in five minutes of the marine box chronometer. This is the centrifugal clock on the station of Zevenboompjes.

A Table of comparison between the Chronometer and centrifugal Clock at Zevenboompjes.

Chronom. Sexages.	Conical Decimal.	Chron. Sexages.	Conical Decimal.	Chron. Sexages.	Conical Decimal.
′	′	′	′	′	′
5	348,31	5	348,23	5	348,17
	348,10	5	347,85		348,21
	347,85		348,19		348,25
	348,37		348,32		348,19
	348,31		348,28		347,70
	348,70		348,18		348,25
	347,52		347,84		348,18
	348,03		348,26		348,25
	347,78		348,24		348,14
	347,84		348,28		348,42
	348,06		348,16		348,13
	347,94		348,10		348,12
	348,09		348,65		348,22
	348,04		348,37		348,39
	348,08		348,47		348,39
	347,87		348,31		348,34
	348,15		348,18		348,37
	348,10		348,29		348,25
	347,05		348,42		348,30
	347,98		348,23		348,13
	348,13		348,26		348,56
	348,31		348,56		348,21
	348,04		348,34		348,18
	348,20		348,22		348,04
	348,24		348,28		348,31
	348,04		348,36		348,40
	347,93		348,55		348,34
	348,21		348,40		348,10
	347,94		347,63		348,18
	348,15		347,32		348,23

Thus, in 445 sexagesimal minutes by the box chronometer of Zevenboompjes, the index of the centrifugal clock of the same station made 30986,83 turns, and thus one sexagesimal minute on the chronometer was equal to 69″,63 decimal seconds of the conical pendulum clock. The table shows that the conical pendulum clock, during the course of the experiments, did not alter its rate with respect of the chronometer. The comparisons between the clock and the chronometer were made at various times of the day, and immediately before and after the experiments.

At the other station of Zevenboompjes, the box chronometer and conical pendulum clock were frequently compared before, during, and after the experiments. The following table shows that the accuracy of the conical pendulum clock was quite sufficient for the purpose in view.

A Table of comparison, showing how many decimal seconds the conical pendulum clock of the station of Kooltjesberg beat in one sexagesimal minute by a chronometer.

Sexagesimal minute.	Decimal seconds.	Sexagesimal minute.	Decimal seconds.
′	″	′	″
1	69,33	1	69,39
1	69,44		69,68
	69,35		69,56
	69,64		69,22
	69,38		69,44
	69,78		69,45
	69,38		69,23
	69,44		69,47
	69,22		69,70
	69,30		69,38
	69,32		

Thus, in 21 sexagesimal minutes the index of the centrifugal pendulum at Kooltjesberg made 1458,″10 beats.

Thus, one sexagesimal minute on the chronometer of Kooltjesberg, equal to 69,″433 turns of the index of the centrifugal clock of the same station.

SECTION VI.

Experiments on the velocity of sound on the 27th June, 1823, compared with theory.

Having thus far stated the means by which the distance between the stations of Kooltjesberg and Zevenboompjes was ascertained, and the rate determined of the clocks by which the velocity of sound was measured, I will now proceed to give the experiments which were made on the 27th of June, and compare the result with theory. The following table contains the time which sound employed to travel over the basis on the 27th of June, when 22 shots were simultaneously fired, and equally seen and heard on both stations. The first column of this table shows the number of the shot, the second the time which sound employed to travel from Kooltjesberg to Zevenboompjes, as observed on the latter station, and the third column the time which sound employed to come from Zevenboompjes to Kooltjesberg, also observed on the latter place.

Experiments on the velocity of sound, made the 27th June, 1823.

I.	II. Sound travelled from Kooltjesberg to Zevenboompjes.	III. Sound travelled from Zevenboompjes to Kooltjesberg.
	″	″
1	52,90	51,17
3	52,69	50,89
4	52,71	50,68
5	52,92	50,80
6	52,84	50,86
7	53,04	50,89
8	52,89	51,01
9	52,79	51,00
11	52,83	50,99
12	52,77	50,96
13	52,79	51,10
14	52,99	51,07
16	52,90	51,08
17	52,64	51,28
18	52,90	51,21
19	52,87	51,18
20	52,92	51,33
22	52,91	51,38
23	52,64	51,35
24	52,57	51,32
25	52,90	51,14
26	52,96	51,01
	1162,37	1123,70
	1123,70	
	2286,07	

and $\frac{2286,07}{44} = 51,''96.$

Thus, taking the mean of all the observations, we have the velocity with which sound travelled along our basis free of

the accelerating or retarding effect of wind, on the 27th of June, 1823, equal to 51″,96. And as the basis was equal to 17669,28 metres, or 9664,7044 fathoms, we have the velocity of sound, found by experiment as above, equal to 340,06 metres, or 1116,032 English feet per second.

Now, whilst these 22 shots were fired, the mean temperature of the air was

$$\left.\begin{array}{lcl} \text{At Zevenboompjes} & = & 11^\circ{,}21 \\ \text{Kooltjesberg} & = & 11\ \ 11 \\ \hline \text{Mean temperature on both stations} & & 11\ \ 16 \end{array}\right\} \text{Of centigrade scale.} \quad = t$$

The mean altitude of barometer corrected of the effect of capillarity, and reduced to the temperature of 0° of centrigrade scale, was as follows:

Station of Zevenboompjes . .	$0^m{,}7439$
Kooltjesberg	0 7456
Mean altitude of barometer .	0 74475 $= p$.

The mean tension of aqueous vapour in the atmosphere, as determined by Mr. Daniell's hygrometer, was at

Station of Zevenboompjes	= 0,00901235 metres.
Kooltjesberg	= 0,00949378
Mean tension of aqueous vapour,	0,00925307 $= f$.

The effect of gravity, calculated for mean latitude of Amersfoort and Naarden, by the formula

$$g = (g)\ (1 - 0{,}002837 \cos.\ 2\ l)$$

$$= \frac{9808{,}8}{1{,}000378864} \left\{ 1 - 0{,}002837 \cos.\ 2 \left\{ 52^\circ\ 13'\ 33''\ 35 \right\} \right\}$$

$$g = 9812{,}03 = \text{effect of gravity in lat. } 52^\circ\ 13'\ 33'',35.$$

The ratio of the specific heat of the air when the volume is constant, to the specific heat of air at a constant pressure, or $\frac{c'}{c}$, is, according to the experiments of GAY LUSSAC and WELTER, equal to $1{,}3748 = \frac{c'}{c}.$

In Sir ISAAC NEWTON's formula $\sqrt{\frac{g\,p}{D}}$, by which the velocity of sound is expressed, D is the density of air, that of mercury being taken for unit.

By BIOT's and ARAGO's experiments, the density of perfectly dry air was found at $0^{m},76$ barometrical pressure to be equal to unity divided by 10466,82.

But when the barometrical pressure alters and becomes p, and the temperature becomes t, we have by the law of MARIOTTE

$$D = \frac{p}{10466{,}82 \times 0^{m},76 \left\{ 1 + t . 0{,}00375 \right\}}$$

And introducing into this formula the correction for the aqueous vapour existing in the air, and calling F the tension of aqueous vapour existing in the air, we find

$$D = \frac{p - \frac{3}{8}\,F}{10466{,}82 \times 0^{m},76 \left\{ 1 + t . 0{,}00375 \right\}}$$

This value of D being substituted in Sir ISAAC's formula, we have the velocity of sound by theory

$$V = \sqrt{\frac{g\,p}{D}} = \sqrt{\frac{g\,p . 10466{,}82 \times 0^{m},76 \left\{ 1 + t . 0{,}00375 \right\}}{p - \frac{3}{8}\,F}}$$

$$= \sqrt{\left\{ 10466{,}82 \times 0^{m},76 \left\{ 1 + t . 0{,}00375 \right\} \right\} \frac{g\,p}{p - \frac{3}{8}\,F}}$$

According to LAPLACE, this formula must be multiplied by the square root of the ratio between the specific heat of air at

a constant volume, and the specific heat of air at a constant pressure. Thus the final formula for the velocity of sound, given by theory, is

$$V = \sqrt{\left\{10466{,}82 \times 0^{m}{,}76 \left\{1 + t \,.\, 0{,}03375\right\}\right\} \frac{g\,p}{p - \frac{3}{8}F}} \times \sqrt{\frac{c'}{c}}$$

Substituting in this formula the quantities stated above, theory gives the velocity of sound for the state of the atmosphere on the 27th of June, 1823, when the experiments were made, $V = 335{,}14$ metres, or 1099,885 English feet; but the velocity as obtained by experiment was $340{,}06 = 1116{,}032$ feet.

Difference between theory and experiment the 27th of June, 4,92 metres = 16,147 feet.

SECTION VII.

Experiments on the velocity of sound on 28th of June, 1823, compared with theory.

On the 28th of June, 1823, fourteen simultaneous shots were equally seen and heard on both stations; the following table contains the results.

	Number of shots.	Sound travelled from Kooltjesberg to Zevenboompjes in	Sound travelled from Zevenboompjes to Kooltjesberg in
		″	″
	3	51,81	52,12
	4	51,94	52,10
	5	51,77	51,28
	6	51,98	52,51
	7	52,17	52,46
	8	52,15	52,28
	9	52,25	53,10
	10	52,18	50,17
	12	52,40	52,19
	14	52,27	52,62
	15	52,27	51,66
	17	52,23	51,52
	18	52,49	51,99
	19	52,56	51,60
Sum		730,47	727,60

The mean result by experiment on the 28th of June, 1823, is $\frac{730'',47+727'',60}{28} = 52,07$, in which time sound travelled along the basis of 17669,28 metres, or 57988,2264

English feet. Thus, the mean velocity of sound on the 28th of June in 1″, is 339,34 metres = 1113,669 English feet.

The mean temperature, when these experiments were made, was at

Zevenboompjes - - -	11°,07	Centigrade scale.
Kooltjesberg - - - -	11 36	
Mean temperature - -	11 215 = t	

Mean height of the barometer, corrected for capillarity, and reduced to 0° of centrigrade scale,

Zevenboompjes - - - -	0,7476 metres.
Kooltjesberg - - - - -	0,7487
Mean barometer, or $p =$	0,74815
Mean tension of aqueous vapour by Mr. DANIELL's hygrometer - - F =	0,00840465

These quantities being substituted in the formula, we have the velocity of sound, by theory, on the 28th of June, 1823, V = 335,^{m}10 metres = 1099,753 English feet; by experiment, 339^{m},34 metres = 1113,669 feet.

Difference between theory and experiment 4,24 metres = 13,916 feet.

Thus it appears by the experiments both of the 27th and 28th of June, that sound travels faster than its theorètical calculation.

The 27th of June, difference of experiment and theory 4^{m},92
28th of June - - - - 4^{m},24

The difference between the experiments of 27th and 28th of June, is but of 0^{m},62, or 2,3629 feet; that is about $\frac{1}{472}$ of the mean result of the experiments of both days.

The French philosophers found a difference between their experiments of 23d and 24th of June, 1822, of $\frac{1}{90}$. But the difference of $\frac{1}{472}$, which we obtained, if we reduce the observations of both days to what they would have been in perfectly dry air, and in temperature of 0° *cent.* is still remarkably lessened. The formula by which the velocity of sound in given hygrometrical circumstances, and a given temperature of the air, is reduced to what it would be in dry air of 0° *cent.* temperature, calling U′ the velocity of sound in dry air of 0° temperature; U the velocity of sound at a tension of aqueous vapour = F, is as follows:

$$U' = \frac{U}{\sqrt{\left\{1 + 0{,}00375\, t\right\}} \times \sqrt{\left\{1 - 0{,}37651\right\} \frac{F}{p}}}.$$

The 27th of June, 1823, we had

$U = 340^{m},06 = 1116{,}032$ English feet

$t = 11°,16$ *cent.*

$F = 0{,}00925307$

$p = 0{,}74475$ metres.

Substituting these quantities in the formula, we have

$U' = 332^{m},38 = 1090{,}827$ English feet.

The 28th June, 1823, we had

$U = 339^{m},34 = 1113{,}669$ feet

$t = 11°,215$

$F = 0{,}00840465$;

which being substituted in the formula, we have

$U' = 331^{m},72 = 1088{,}661$ English feet.

Thus the difference between the observations of both days, when reduced to dry air, and 0° *cent.* is $0^{m},66 = 2{,}166$ feet; or $\frac{1}{503}$ of the mean of the observations of both days. It appears also, that by our experiments of the 27th and 28th

of June, 1823, the mean velocity of sound in air perfectly dry, and at 0° temperature, was $332^{m},05 = 1089,744$ feet in a second.

SECTION VIII.

Experiments on the 25th of June, when the shots were not reciprocal.

The following experiments will I trust prove, that in experiments on the velocity of sound, such observations can only be relied on in which the shots on both stations were reciprocal, that is fell within the same second in both places, and were equally heard and seen on both stations. The 25th of June, the cannon fired at Zevenboompjes was not heard at Kooltjesberg, but at Zevenboompjes the report of the guns fired at the other place was distinctly perceived. The following table shows the time preterlapsed between the light and report, as observed at Zevenboompjes.

	Number of Shots.	Time between light and report.	
1823. 25th June.	1.	52,31″	Observations at the station of Zevenboompjes, guns fired at the station of Kooltjesberg.
	2.	52,59	
	4.	52,47	
	7.	52,20	
	8.	52,47	
	10.	52,17	
	12.	52,27	
	14.	52,52	
	15.	52,54	
	16.	52,43	
	17.	51,91	
	19.	52,50	

Sum 628,39, which being divided by twelve, the number of observations, gives the passage of

sound along the basis $52'',37$. Thus the mean velocity in $1'$ was $337^{m},39 = 1107,268$ feet.

The mean temperature at the time of these experiments

$$\left.\begin{array}{lr} \text{at Zevenboompjes} & 7^{\circ},\ 41 \\ \text{Kooltjesberg} & 8\ ,\ 54 \\ \text{Mean temperature of the air,} & 7\ ,975 = t \end{array}\right\} \text{centigrade}$$

Mean height of the barometer corrected for capillarity, and at 0° *cent.*

at Zevenboompjes	$0^{m},7522$
Kooltjesberg	0 ,7538
Mean barometer - - -	0 ,7530 $= p$

Mean tension of aqueous vapour in the air,

at Zevenboompjes	0,00737444
at Kooltjesberg -	0,00706966.
Mean tension - -	0,00722205 $=$ F,

which quantities substituted in the formula, we have for temperature 0° *cent.* and in perfectly dry air the velocity of sound $U' = 331,85$ metres $= 1089,087$ feet.

SECTION IX.

Experiments of 26th of June, 1823, when the shots were not reciprocal.

The 26th of June, the following shots were seen and heard at Kooltjesberg and fired at Zevenboompjes, but no shots from the first station were heard at the latter.

	Number of Shots.	Time between light and report.	
	1.	$''$ 50,20	Guns fired at Zevenboompjes, heard and seen at Kooltjesberg.
	2.	50,80	
	3.	51,44	
	4.	52,20	
	5.	51.10	
	9.	50,11	
	11.	50,99	
	12.	50,81	
	13.	51,00	
	14.	51,01	
	16.	51,12	
Total of 12 shots		560,78	The mean of which is $50''98$

which gives a velocity of 346,59 metres = 1137,134 feet in $1''$.

The temperature was at that time

at Zevenboompjes $11°, 57$

Kooltjesberg $12, 54$

Mean temperature . . $12,055 = t.$

Mean atmospheric pressure at Zevenboompjes $0^{m},7493$.
Mean atmospheric pressure at Kooltjesberg $0^{m},7512$.
Mean pressure of atmosphere $0,75025 = p$.
Mean tension of aqueous vapour at Zevenboompjes 0,00892922
at Kooltjesberg 0,01011376
Mean tension of aqueous vapour . $0,00952149 = F$

Calculating by this datum we shall have the observed velocity of sound in $1''$ reduced to dry air and $0°$ temperature $V = 338^{m},20 = 1109,927$ feet; but the experiments of the 25th gave $V' = 331,^{m}85 = 1089,087$ feet. Difference $6^{m},35 = 20,840$ feet per $1''$ between the experiments of the 25th and 26th of June, in which the shots were not reciprocal. This difference is about $\frac{1}{53}$ of the mean of both observations. But the 27th and 28th of June, when the shots were reciprocal, the difference between the results of both days were only $0^{m},66 = 2,166$ feet, that is about $\frac{1}{503}$ of the mean result of the observations.

From the comparison of these results we may safely infer, that only such shots will answer the purpose for which these experiments are made, which are exactly fired at the same instant on both stations.

It is in this respect, I imagine, that our experiment may claim some attention, as the very great care and ability of our artillery men enabled us have the guns fired within the interval of one second.

A Table showing the results of experiments on the velocity of sound as observed by different philosophers.

Names of Observers.	Time when made.	Country where made.	Length of basis. metres.	Velocity of Sound per Second in metres.	
Mersenne		France		448	1
Florentine Philosophers	1660	Italy	1800	361	2
Walker	1698	England	800	398	3
Cassini, Huigens, &c.		France	2105	351	4
Flamsteed and Halley		England	5000	348	5
Derham	1704 & 1705	England	1600 à 2000	348	6
French Academians	1738	France	22913 & 28526	332,93 at 0° c	7
Blanconi	1740	Italy	24000	318	8
La Condamine	1740	Quito	20543	339	9
La Condamine	1744	Cayenne	39429	358	10
T. F. Mayer	1778	Germany	1040	336,86	11
G. E. Muller	1791	Germany	2600	338	12
Epinoza and Banza	1794	Chili	16345	356,14 at 0°	13
Benzenberg	1809	Germany	9072	333,07 at 0°	14
Arago, Mathieu Prony	1822	France	18612	331,05 at 0°	15
Moll, Van Beek, and Kuytenbrouwer	1823	Netherlands	17669,28	332,05 at 0° and dry air.	16

1. Mersenne de Arte Ballistica Prop. 39.
2. Tentamina Experim. Acad. del. Cimento, L. B. 1738, Part II. p. 116.
3. Philos. Trans. 1698, No. 247.
4. Duhamel, Hist. Acad. Reg. L. II. Sect. 3, Cap. II.
5. Philos. Trans. 1708 and 1709.
6. Ibid, ibid.
7. Mem. de l'Academie des Sciences, 1738 and 1739.
8. Comment. Bononienses, Vol. II. p. 365.
9. La Condamine Introduction Historique, &c. 1751, p. 98.
10. Mem. de l'Acad Royale des Sciences, 1745, p. 488.
11. J. T. Mayer, Praktische Géomêtrie Göttingen, 1792, B. 1. p. 166.
12. Muller, Götting. Gelehrt. Anzeige, 1791, St. 159 & Voigts Magazin, &c. B. 8, St. 1. p. 170.
13. Annales de Chimie. et de Phys. T. VII. p. 93.
14. Gilbert's Annalen, neue Folge, B. V. p. 383.
15. Connaissance des Tems. 1825, p. 361.

Communicated by Captain H. Kater, *F. R. S.*